Mean Value Theorem

1. The Mean Value Theorem guarantees the existence of special point(s) on the graph of $y = x^3 + x + 7$ on $0 < x < 3$. What are the coordinates of the point(s)?

2. Let f be the function given by $f(x) = x + \frac{1}{x}$. What are all values c that satisfy the conclusion of the Mean Value Theorem on the closed interval $[1, 2]$?

3. Consider the piecewise function $f(x) = \begin{cases} x + 5, & x < -2 \\ x^2 + 2x + 3, & x \geq -2 \end{cases}$.

   Find the average rate of change of f on the interval $-4 \leq x \leq 3$. Explain why the Mean Value Theorem does not guarantee a point $c, -4 < c < 3$, for which $f'(c)$ is equal to that average rate of change.